Spatial Statistics

Summer semester 2020

Practical 1st Order Characteristics

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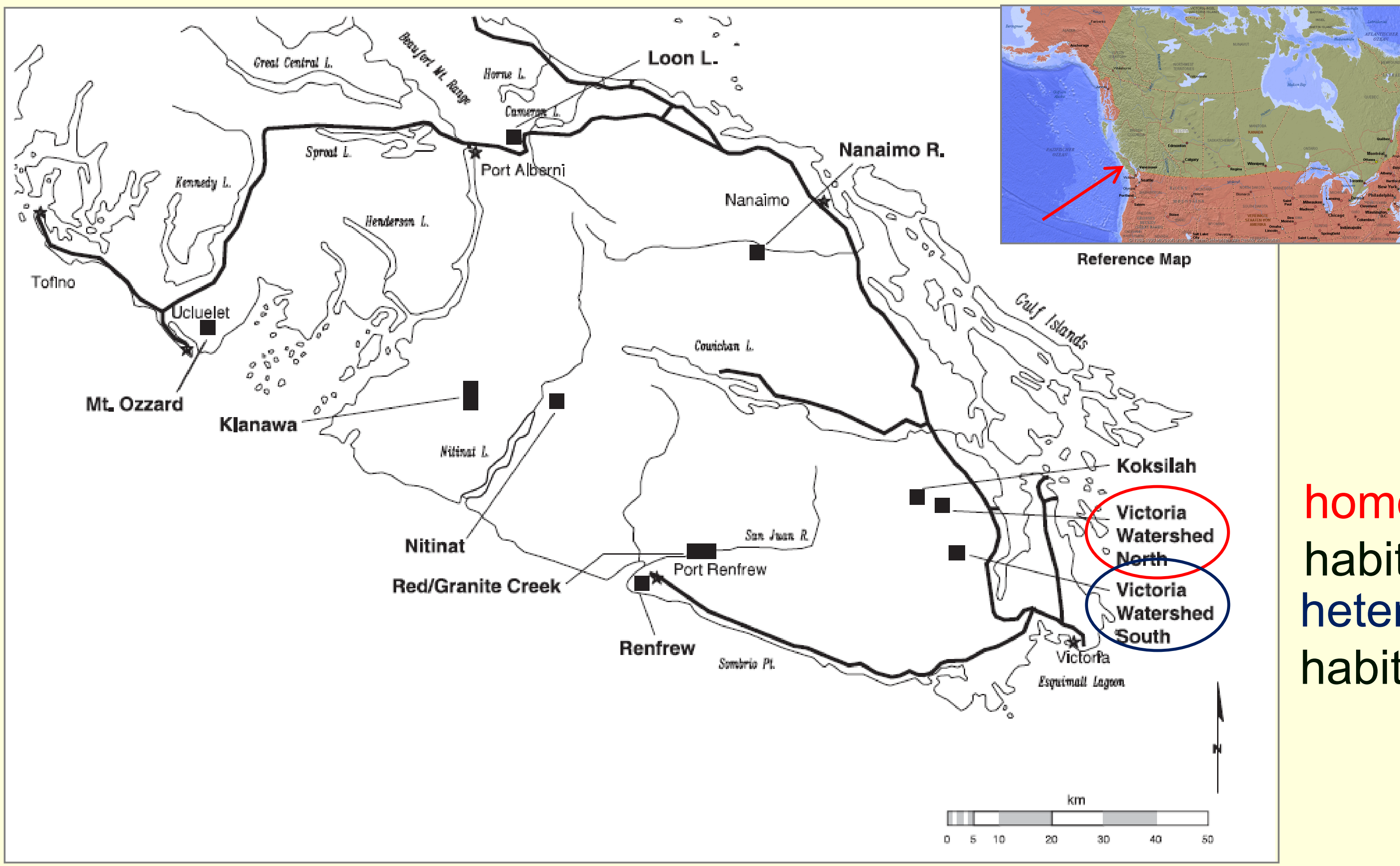
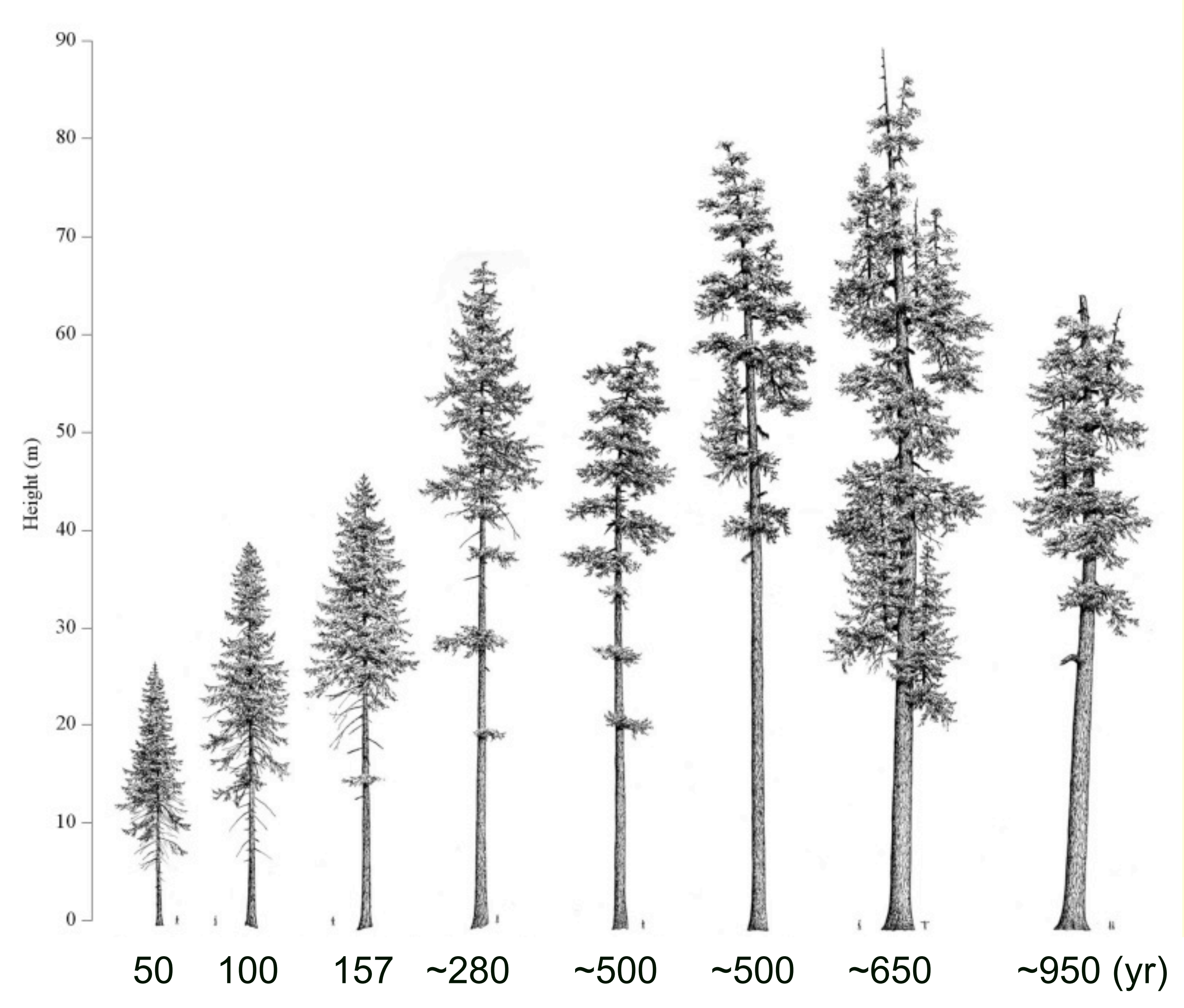
Coastal temperate forest on south-eastern Vancouver Island



homogeneous habitat

heterogeneous habitat

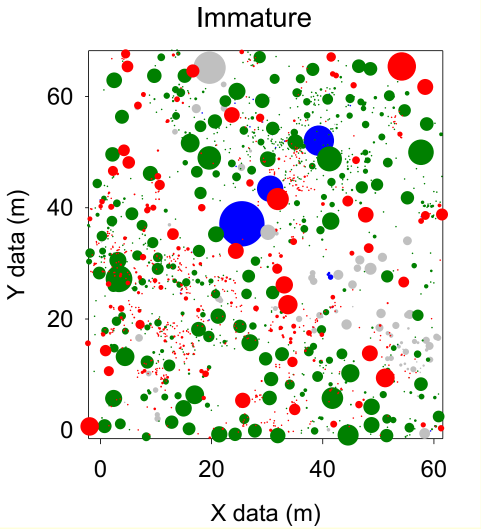
Aging pioneer Douglas-fir after stand replacing fire



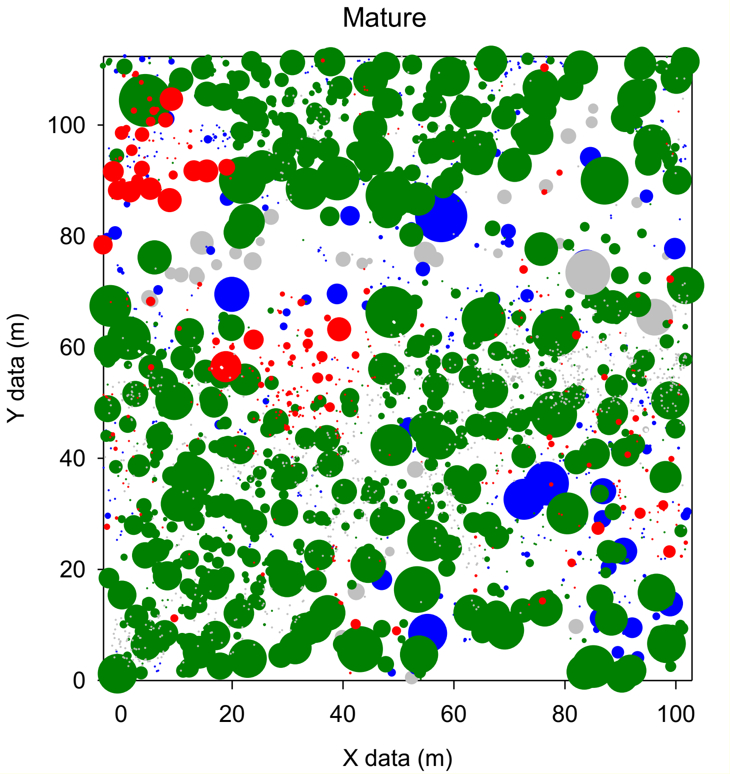
Shade-tolerant, late-successional western hemlock (left) and western redcedar

Chronosequence

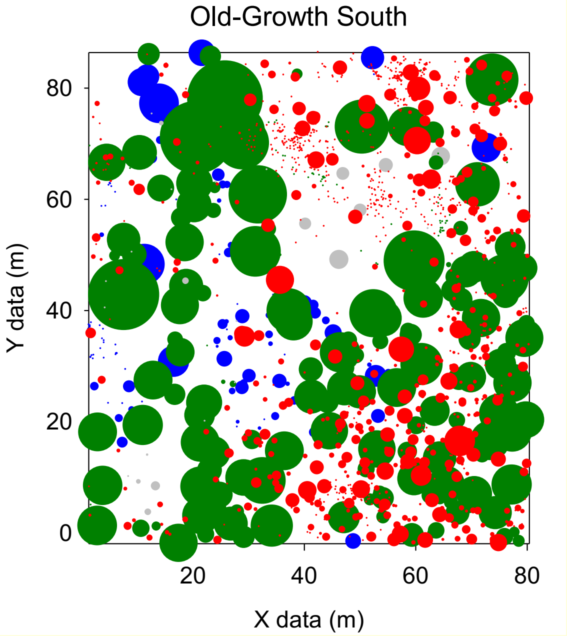
39 yr



109 yr



254 yr



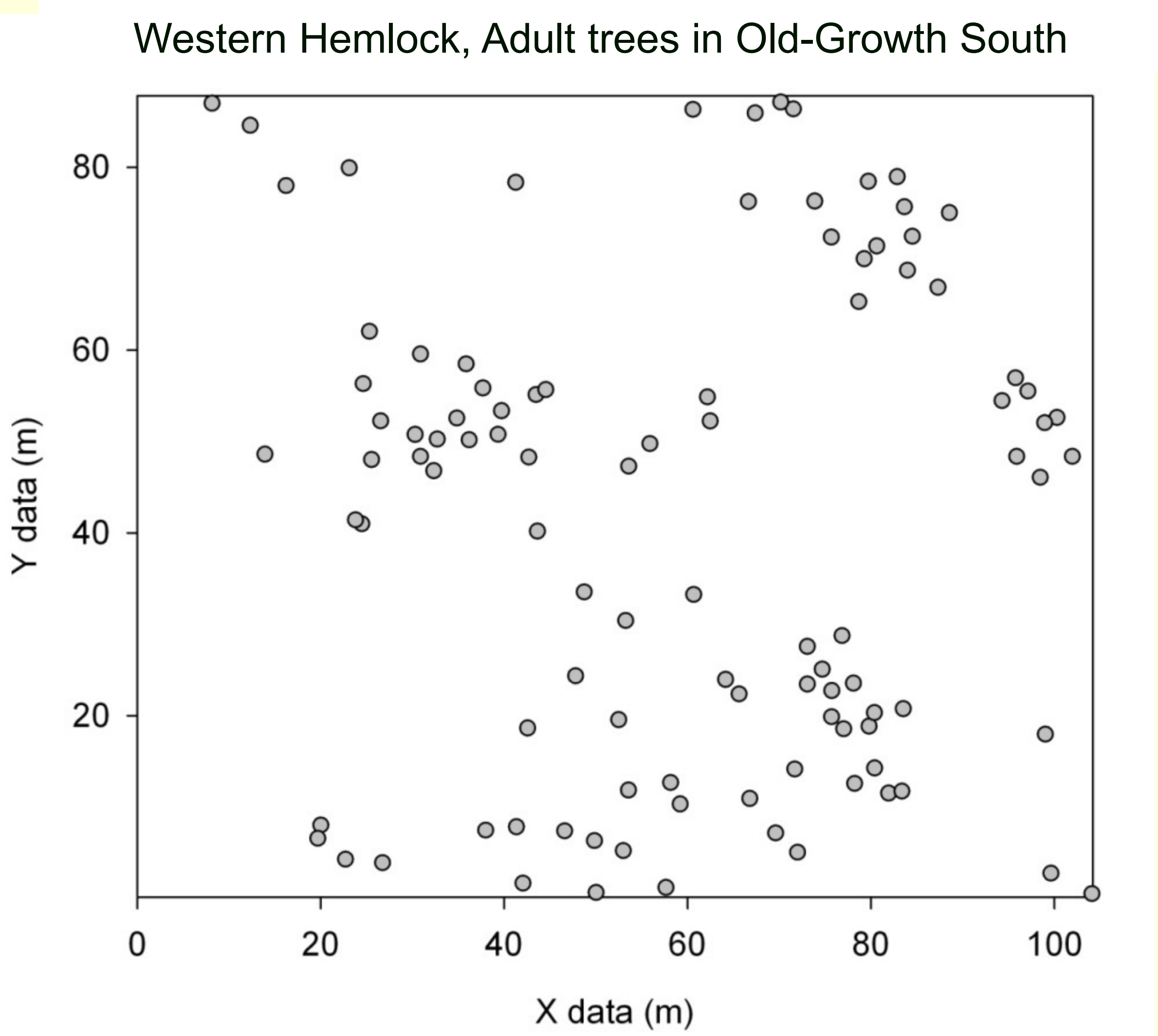
- Douglas-fir
- western hemlock
- western redcedar
- all others

for all trees measured:

- x-y-coordinates, DBH (diameter at breast height)
- height, status

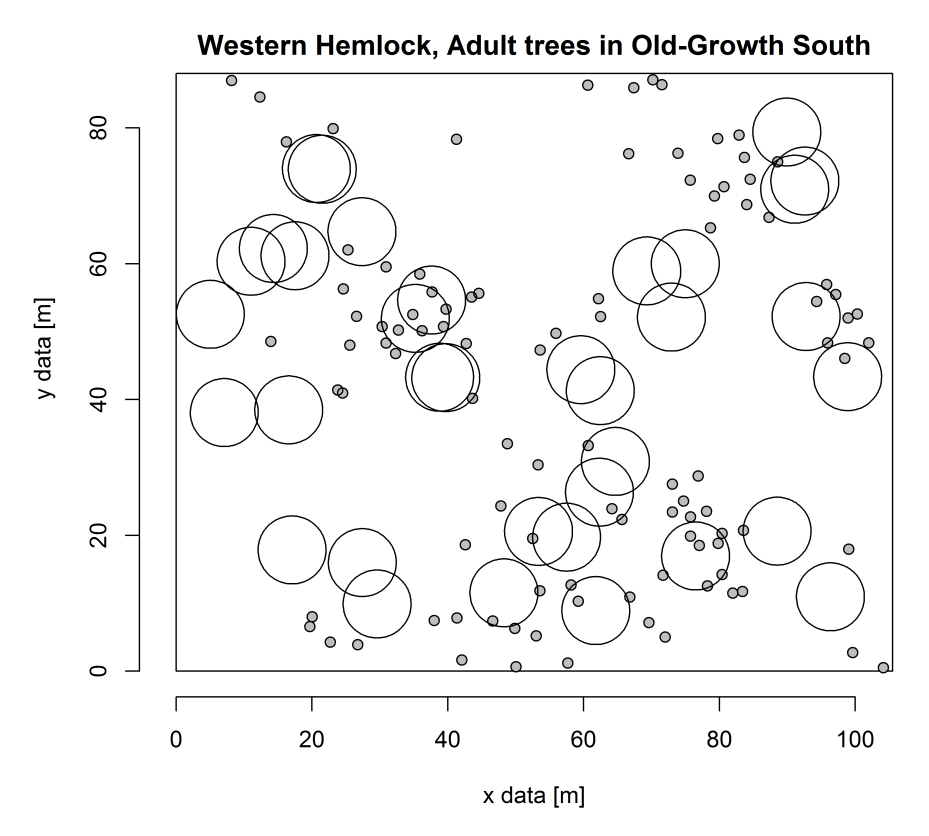
bubble size ~ DBH

1st order properties – comparing theoretical Poisson distribution against empirical data



1st order properties – comparing theoretical Poisson distribution against empirical data

- ⇨ estimate intensity λ by randomly throwing coin (35×)
- ⇨ coin = “*moving window*“ (two groups: 1 Cent & 2 Euro)
- ⇨ λ = absolute number events in samples / number of samples
- ⇨ e.g.: 32 events in m = 35 samples of fixed size
  λ = 0.914 events/area

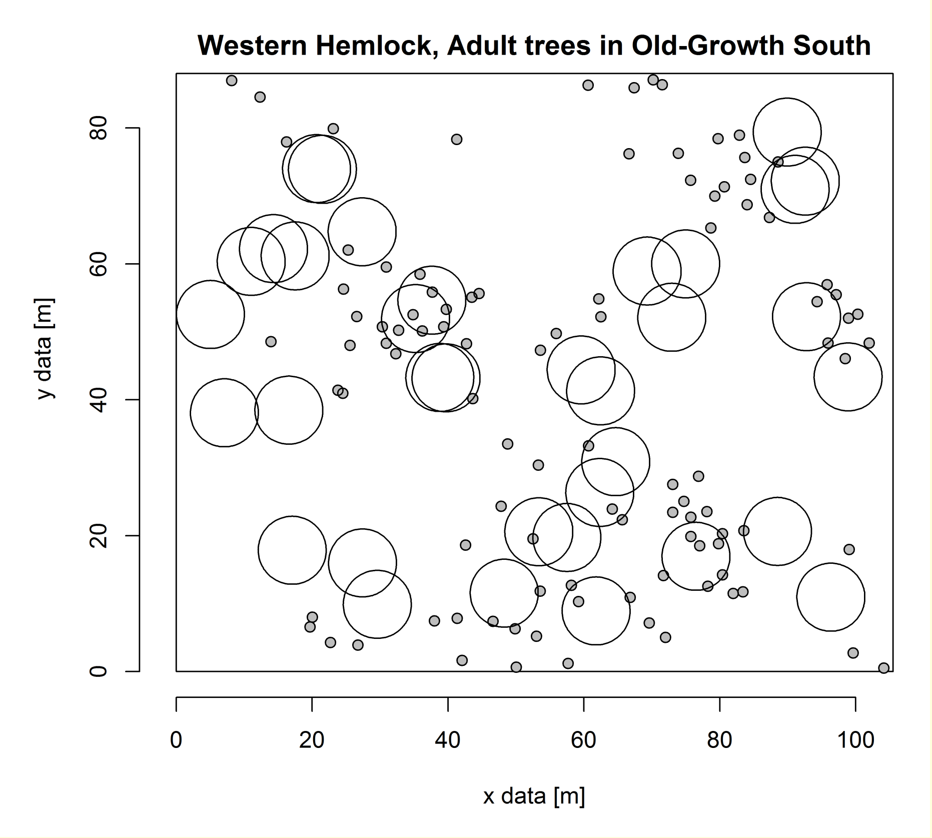


1^{st} order properties – comparing theoretical Poisson distribution against empirical data

$$P_\lambda(n) = \frac{\lambda^n}{n!} e^{-\lambda}$$

- $P_\lambda(n)$ = probability to find *n* trees within a circle (of area *B*=1 unit) for the estimated intensity *λ*

- $P_{0.914}(0) = 0.401 \quad = 40.1\%$
- $P_{0.914}(1) = 0.366$
- $P_{0.914}(2) = 0.168$
- $P_{0.914}(3) = 0.051$
- $P_{0.914}(4) = 0.012$
- $P_{0.914}(5) = 0.002$
- $P_{0.914}(6) = 0.000 \quad = 0.03\%$



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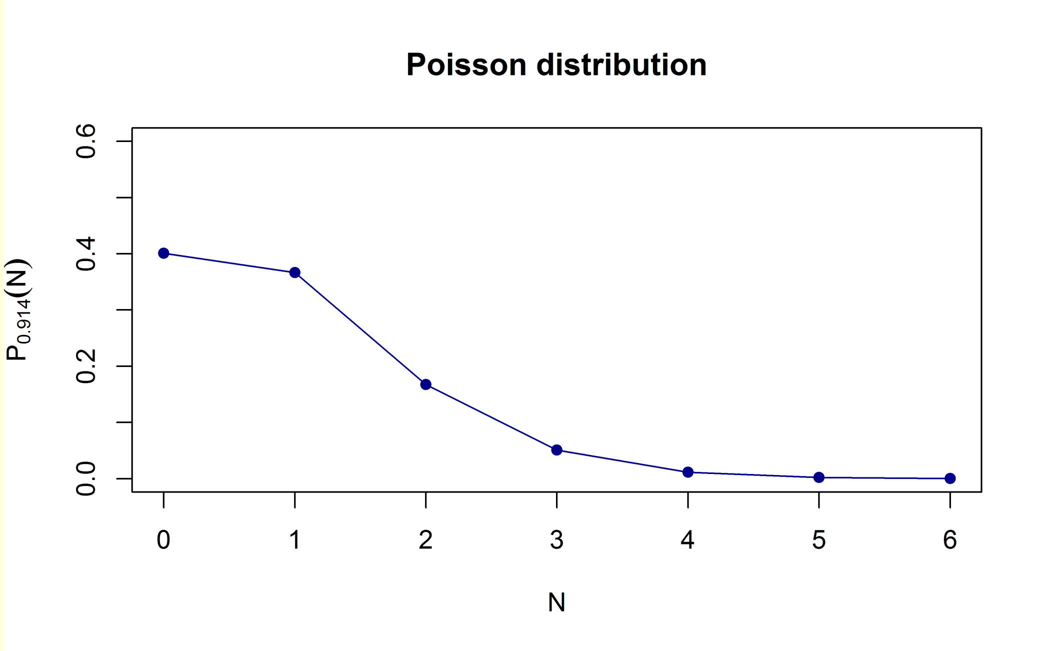
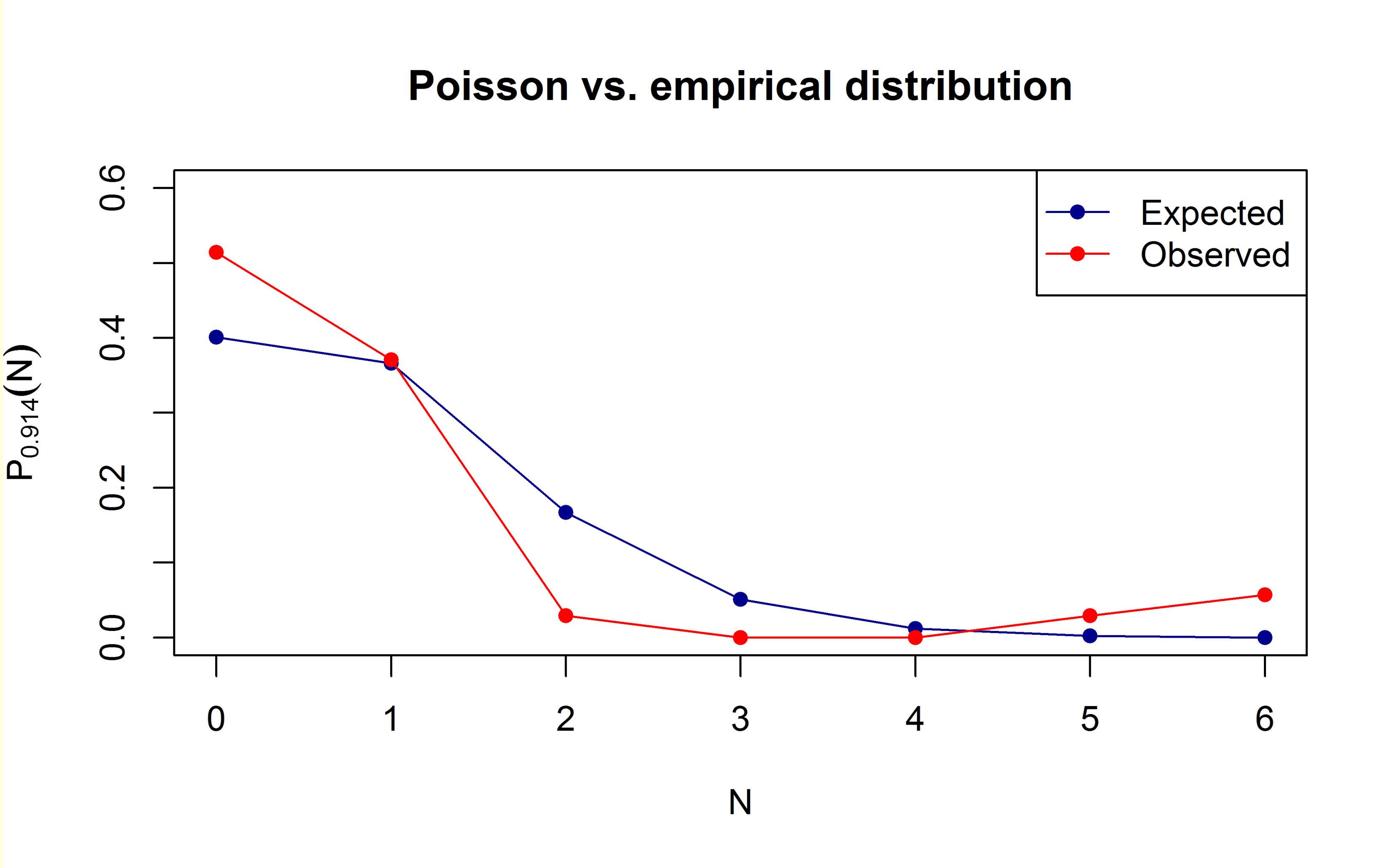


1st order properties – comparing theoretical Poisson distribution against empirical data

Number (n) of events in sample category	$P_\lambda(n)$	Expected number of discoveries in sample category $= 35 \times P_\lambda(n)$	Observed number of discoveries x in sample category	Observed fraction of discoveries in sample category
0	0.401	14.04	18	0.514
1	0.366	12.81	13	0.371
2	0.167	5.85	1	0.029
3	0.051	1.78	0	0.000
4	0.012	0.42	0	0.000
5	0.002	0.07	1	0.029
6	0.000	0.00	2	0.057

1st order properties – comparing theoretical Poisson distribution against empirical data



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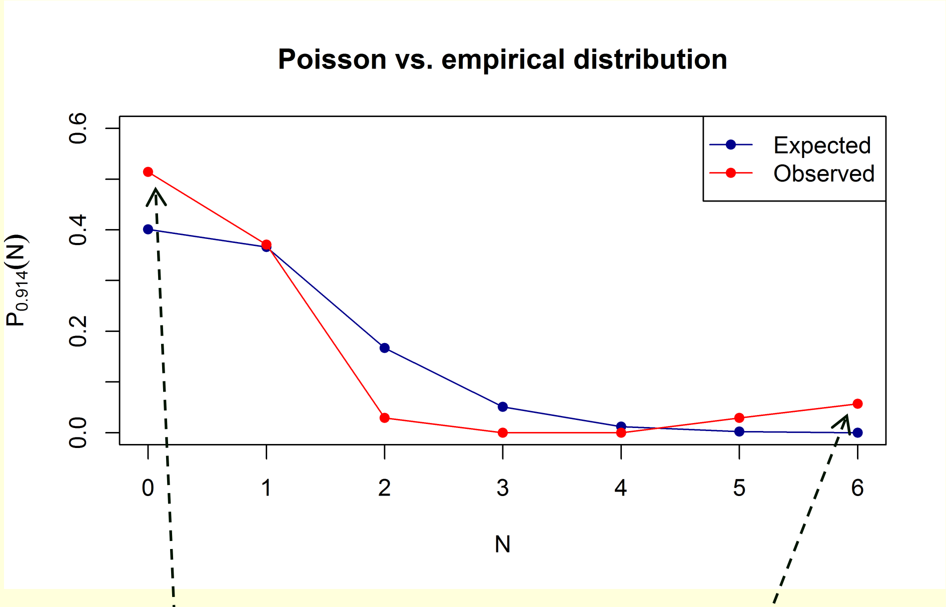

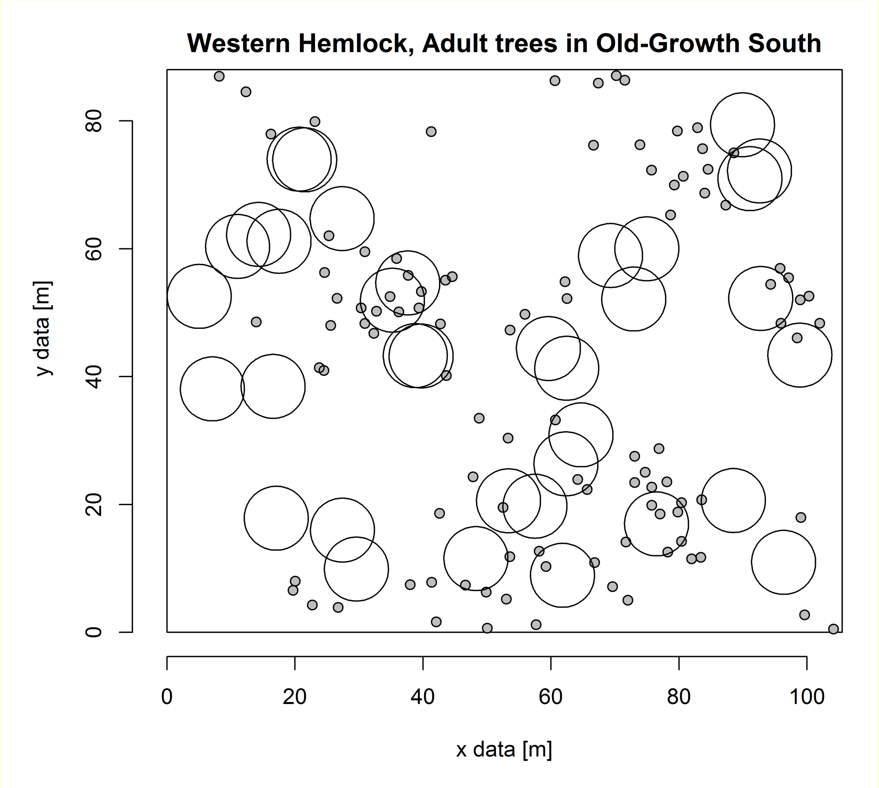

„holes“

„clustering“

1st order properties – comparing theoretical Poisson distribution against empirical data

…..with chi-square test (because of small sample size)

H_0: Western hemlock adult trees are randomly distributed.

H_1: Western hemlock adult trees are not randomly distributed.

1st order properties – comparing theoretical Poisson distribution against empirical data

…..with chi-square test (because of small sample size)

H_0: Western hemlock adult trees are randomly distributed.

H_1: Western hemlock adult trees are not randomly distributed.

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

O = Observed value

E = Expected value

1st order properties – comparing theoretical Poisson distribution against empirical data

Number (n) of events in sample category	Expected number of discoveries in sample category $= 35 \times P_{\lambda}(n)$	Number of observed discoveries in sample category	chi-square $\frac{(O-E)^2}{E}$
0	14.04	18	1.117
1	12.81	13	0.003
2	5.85	1	
3	1.78	0	
4	0.42	0	
5	0.07	1	
6	0.00	2	
>3 (2–6)	8.21	5	2.090

all expected values should be 3 or more

$\sum = 3.210$

d.f. = k-2 = 1; critical value $\chi^2_{0.95,1d.f.}$ = 3.841 (> 3.210) **-> do not reject H_0**

1st order properties – comparing theoretical Poisson distribution against empirical data

Number (n) of events in sample category	Expected number of discoveries in sample category $= 35 \times P_\lambda(n)$	Number of observed discoveries in sample category	chi-square $\frac{(O-E)^2}{E}$
0	14.04	18	1.117
1	12.81	13	0.003
2	5.85	1	4.021
3 } >1	1.78 } 2.27	0 } 4	} 0.235
4	0.42	0	
5	0.07	1	
6	0.00	2	

all expected values should be 3 or more

$\sum = 5.376$

d.f. = k-2 = 2; critical value $\chi^2_{0.95,2d.f.}$ = 5.991 (> 5.376) **-> do not reject H_0**

2nd order properties

Is the pattern is not CSR, but is it clustered or regular?

under CSR the variance $\sigma^2 = \overline{\mathrm{x}}$ Index of dispersion $I = \frac{s^2}{\mu} = \frac{\frac{1}{m-1}\sum_{\mathrm{i}=1}^{m}(\mathrm{x}_\mathrm{i} - \overline{\mathrm{x}})^2}{\frac{1}{m}\sum_{i=1}^{m} x_i}$

2nd order properties

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if *I* > 1, pattern is clustered

if *I* < 1, pattern is regular

I for Western hemlock adults = 2.726 **-> trees are clustered (n.s.)**

(at scale of moving window)